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Towards an Integrated Framework for Development and Environment Policy: The Dynamics of Environmental Kuznets Curves

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Summary. — Environmental Kuznets curves (EKCs) have recently received considerable attention in development and environment policy. But little is known on how the interaction between ecological and economic factors may result in an EKC or other qualitatively different outcomes. We introduce a restricted class of vector valued differential equations for representing the evolution of socioeconomic and environmental variables which influence or are influenced by the process of economic development. Our model includes not only the complete path of each variable over time but also the highly critical interactions among multiple variables. We demonstrate analytically and numerically that among a multiplicity of possible outcomes, an inverted-U pattern can only be obtained under specific circumstances, and requires attention to the multiple factors which form the economic–environmental system, rather than a single dominant one. © 2001 Elsevier Science Ltd. All rights reserved.

Key words — development and environment policy, environmental change, economic growth, environmental Kuznets curves, system dynamics

1. INTRODUCTION

A concept crystallizing in the development and environment literature is the notion that socioeconomic or environmental measures follow predictable paths associated with growing per-capita income (see, for example, Beckerman, 1992; Grossman & Krueger, 1993, 1995; Holtz-Eakin & Selden, 1992; Kuznets, 1955; Selden & Song, 1994; Shafik, 1994; Shafik & Bandyopadhay, 1992). In this growing trend, one specific relationship between economic growth and other socioeconomic or environmental variables has become the focus of increasing attention. This relationship is the "inverted-U" curve in which the variable of interest—inequality or pollution, for instance—first increases and then, after a "turning point," declines as income grows.

Numerous works have reviewed the assumptions and conclusions of the environmental Kuznets curves (EKCs) literature (Arrow *et al.*, 1995; Banuri, 1997; Lopez, 1994; Rock, 1996; Stern, Common, & Barbier,

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1996; Stern, 1998; see also the articles in the October 1997 issue of the *Environment and Development Economics* and May 1998 issue of *Ecological Economics*). Rather than focusing on the conclusions of the "environmental Kuznets curve theory," we provide a short critique of the methodology and assumptions used in arriving at the inverted-U relationship, especially as related to policy formulation. We then use this critique as the basis for the construction of an alternative methodology for policy evaluation.

Our framework considers not only the complete path of each socioeconomic or environmental variable over time but also the critical interactions among the variables. We first state the conceptual foundations of the method and then develop a mathematical formulation. Based on the model, we derive the range of conditions which can result in an inverted-U pattern for the socioeconomic and/or environmental variables in the system. We discuss the important issues in applying the model to policy analysis, and use parameter estimates drawn from published sources of environment and development indicators to analyze its performance and attributes. The analysis demonstrates that among a multiplicity of possible outcomes, an inverted-U pattern can be obtained only under very specific circumstances, and requires attention to the multiple factors which form the economic–environmental system, as raised qualitatively by Arrow *et al.* (1995).

2. REVIEW OF THE EXISTING LITERATURE ON EKC [1]

EKCs are broadly based on the argument that while the initial stages of economic growth are accompanied by increasing pollution, once per-capita income exceeds a threshold, not only does the structure of the economy change, but also people can afford to demand a cleaner environment (Beckerman, 1992; Shafik, 1994; Shafik & Bandyopadhay, 1992). The improvement in environmental quality can be achieved by changing the technological mode of production (de Bruyn, 1997; Han & Chatterjee, 1997) or by exporting the "dirty industry" to low-income countries (Rock, 1996; Suri & Chapman, 1998).

Theoretical arguments for the EKCs have also been supported by empirical research (see Cole, Rayner, & Bates, 1997; Cropper & Griffiths, 1994; Grossman & Krueger, 1993, 1995; Roberts & Grimes, 1997; Selden & Song, 1994; Shafik, 1994; Shafik & Bandyopadhay, 1992). The number of environmental indicators and the data used in these analyses have evolved over time to include various pollutants as well as indirect environmental indicators such as energy use and transportation. The basic approach in many empirical studies has nonetheless changed little from the initial study of Grossman and Krueger (1993). Using data from various countries at various points in time, these works find a mix of relationships between pollution and income, including increasing, decreasing, inverted-U, and inverted-U with reversal at higher incomes.

This diversity of relationships clearly suggests the need for more elaborate models of the underlying phenomena. To address this concern, in a recent study, in addition to income growth, Panayotou (1997) considers the role of income growth *rate*, population density, scale of economy, and a policy variable in determining the income–environment relationship. Agras and Chapman (1999) also add energy prices to the independent variables, and find that income is no longer a determinant of carbon emission.

To improve upon the use of aggregate data, Han and Chatterjee (1997) decompose carbon emissions into its components and consider the change in emissions due to change in the structure as well as in each of its components over time. Although this work provides a dynamic framework for studying the impacts of growth on the environment, it continues to overlook the feedbacks among economic growth, environmental change, and the structure of emissions. Finally, an empirical analysis of the role of structural change by de Bruyn (1997) for SO_2 emissions in (West) Germany and Netherlands illustrates that sulfur emission in these two countries has declined as a result of change in the (technological) mode of production, and not in the structure of the economy.

In addition to analyzing the income–environment trends on their own, another group of studies have considered the factors which are believed to cause an inverted-U pattern: demand for environmental quality and change in the structure of economic production. In formalizing the transition to the low-pollution state, Lopez (1994), McConnell (1997), and Stokey (1998) all provide valuable analyses of the role of regulation and preferences on the emissions profile of polluters. But these works do not consider the ecological attributes of the system (such as the role of accumulated pollution,[2] important environmental thresholds, or the rate of the recovery of the environment from pollution); neither do they address the important feedbacks from the environment to economic activity. We take a step in this formulation.

3. EXAMINATION OF THE EKC METHODOLOGY

Using cross-sectional data[3] of a single environmental variable as an indicator of the development path of each individual country suffers from the following methodological shortcomings:

—Considering the path of a single variable masks the fact that there are often interactions among various economic and environmental phenomena (Arrow *et al.*, 1995; Stern *et al.*, 1996).[4]

—Pooling data from different countries in a single regression is not justified in the presence of differences between countries (without including the variables that generate such differences) as illustrated in Figure 1. Empirical evidence for such differences is given by Moomaw and Tullis (1994) and Moomaw and Unruh (1997) for carbon dioxide emissions and by Bowman (1997) for income inequality and growth. Vincent (1997) finds differences between cross-country analysis and single-country results.[5]

To build and evaluate an alternative method that addresses the above issues, we formulate a framework for development-environment policy that:

—acknowledges the interaction among various socioeconomic and environmental variables and provides a way to track such interactions, and

—explicitly considers the whole duration of the development process and the impact of the circumstances under which a development policy is applied on the outcome.

4. CONCEPTUAL AND MATHEMATICAL FORMULATION OF THE MODEL

(a) *Mathematical notation*

Let $V_1, V_2, \ldots, V_m$ denote a set of m socioeconomic or environmental variables which are measured in a population. For each individual

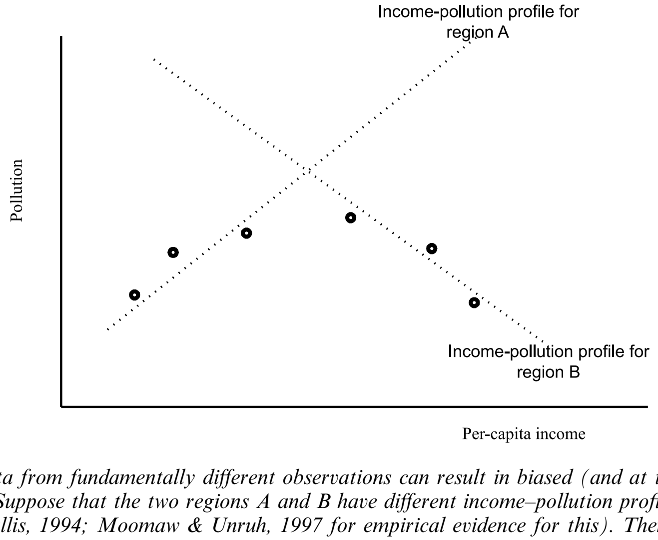

Figure 1. *Pooling data from fundamentally different observations can result in biased (and at times meaningless) regression coefficients. Suppose that the two regions A and B have different income–pollution profiles (see Koop & Tole, 1999; Moomaw & Tullis, 1994; Moomaw & Unruh, 1997 for empirical evidence for this). Then pooling observations from the two regions can result in regression coefficients which do not apply to either region (especially if they are observed at different income ranges as shown in the figure). This is an example of the heterogeneity problem when using panel data. The problem is caused by the fact that additional factor(s)—the structure of the economy or the ecological attributes of the region, for instance—are also important in determining the level of pollution, but have not entered the regression equation causing an omitted variable or omitted fixed-effect bias for the estimators.*

in the population, the measurements at time t constitute an array of size m, defined as

$$\mathbf{V}(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \\ \vdots \\ V_m(t) \end{bmatrix}.$$

Suppose that at any given time the change (or *drift*) in the value of each socioeconomic or environmental variable, V_i, depends on the interactions among all the variables in the system via the following differential equation:

$$\mathrm{d}\mathbf{V}(t) = \mathbf{b}(\mathbf{V}(t), t)\,\mathrm{d}t, \quad \mathbf{V}(t = 0) = \mathbf{V}^0, \tag{1}$$

where $\mathbf{b}(\mathbf{V}(t), t)$ is the drift vector. [6]

The impact of technology or policy on the economic–environmental system is then summarized through the parameters of the drift term (which can change over time). A framework such as that laid out by Stokey (1998), for example, can provide estimates for the parameters which quantify the relationship between income and emissions at any income level.

(b) *A specific form for the drift term*

To obtain a specific form for the drift vector, assume that in each time interval the value of the variable changes by a constant amount, plus a linear combination of the *current values* of all the variables in the system, plus a linear combination of the *changes* in all the variables in the system. Using the above notation for the drift term

$$\mathbf{b}(\mathbf{V}(t), t) = \mathbf{b}_0(t, \mathbf{V}(t)) + \mathbf{B}(t, \mathbf{V}(t))\mathbf{V}(t) + \mathbf{D}(t, \mathbf{V}(t))\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}, \tag{2}$$

where $\mathbf{b}_0$ is the vector of constant changes per unit time, $\mathbf{B}$ is the $(m \times m)$ matrix of coefficients for the effects of current values of the system variables, and $\mathbf{D}$ the $(m \times m)$ matrix of coefficients for the effects of the changes in the values of the system variables. Therefore, the variable dynamics is given by substituting the relationship of Eqn. (2) for $\mathbf{b}(\mathbf{V}(t), t)$ in Eqn. (1)

$$\mathrm{d}\mathbf{V}(t) = \left(\mathbf{b}_0(t, \mathbf{V}(t)) + \mathbf{B}(t, \mathbf{V}(t))\mathbf{V}(t) + \mathbf{D}(t, \mathbf{V}(t))\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}\right)\mathrm{d}t. \tag{3}$$

In adopting this form for the drift term, we assume that at each point in time each variable, V_i, is affected by any other variable in the system in two ways: (1) a proportional component represented as $B_{1i}V_1(t) + B_{2i}V_2(t) + \cdots + B_{mi}V_m(t)$; and (2) a linear component (or linear in the range of values considered in each study) represented as

$$D_{1i}\frac{\mathrm{d}V_1(t)}{\mathrm{d}t} + D_{2i}\frac{\mathrm{d}V_2(t)}{\mathrm{d}t} + \cdots + D_{mi}\frac{\mathrm{d}V_m(t)}{\mathrm{d}t}.$$

The motivation for choosing the above form—the combination of proportional and linear components—for the drift term comes from the observed characteristics of many economic–environmental systems (although other economic–environmental systems may require more complex characterization). The proportional component refers to the relationship in which the "dependent" variable rises (or falls) at a rate proportional to the *current value* of the "independent" variable(s). Therefore, in the relationships represented by this component the "dependent" variable continues to change even when the "independent" variables remain constant. Many environmental effects, such as ecological damage as a result of mining or large-scale irrigation, continue to grow at constant levels of economic activity. The linear component on the other hand, represents situations in which the two variables change simultaneously such as the relationship between air pollution and health.

A simple example may serve to further clarify such a specification of the drift term. Assume that the economic–ecological system of interest includes three variables: food consumption/production (V_1), health (V_2), and the state of the environment (V_3). In a system described by Eqn. (3), in each time interval the value of each variable, V_i, changes by [7]

$$\mathrm{d}(V_i(t)) = \left(b_{0i} + B_{i1}\cdot V_1(t) + B_{i2}\cdot V_2(t) + B_{i3}\cdot V_3(t) + D_{i1}\cdot\frac{\mathrm{d}(V_1(t))}{\mathrm{d}t} + D_{i2}\cdot\frac{\mathrm{d}(V_2(t))}{\mathrm{d}t} + D_{i3}\cdot\frac{\mathrm{d}(V_3(t))}{\mathrm{d}t}\right)\mathrm{d}t, \quad i = 1, 2, 3.$$

In the above system if health improves proportionally with food consumption, $D_{21} > 0$; if the environment is not (directly) dependent on human health, $B_{32} = 0$ and $D_{32} = 0$; or if the current level of food production causes damage to the environment (through pesticide use or soil erosion due to agricultural practices for example) and the damage also rises with increasing agricultural activity $B_{31} > 0$ and $D_{31} > 0$; if an exogenous factor—say new medical technology—improves health by a constant amount in each time period—by 0.1 "health units," for example,—then $b_{0_2} = 0.1$; and so on.

(c) *The outcome of the drift process*

In Appendix A we derive the outcome of the drift process by solving the system of differential equations given by Eqn. (3). We then use this solution to determine under what conditions the output of the drift component would exhibit specific characteristics or shapes—increasing, decreasing, or the inverted-U shape for instance.

The set of conditions in Table 1 in Appendix A—all of which are observed in actual environmental–economic systems—illustrates that in a system in which economy and the environment interact through feedbacks, an inverted-U is only one possible outcome among a multiplicity of relationships, and comes about under specific conditions. This multiplicity of relationships, itself caused by the pair-wise interaction between the components of the system, emphasizes the need for locally-specific analysis of development policies and projects.

5. PERFORMANCE OF THE MODEL: ANALYSIS OF A SIMPLE ECONOMIC–ECOLOGICAL SYSTEM

We used three economic and environmental variables—income, food consumption and/or production, and environmental damage (or pollution)—for analyzing the performance of the model. We obtained estimates of the parameters describing the interaction among these variables from various sources such as the World Bank publications and scholarly articles as summarized in Appendix B. With parameter estimates obtained from different sources and contexts, we emphasize that the purpose of this exercise is not to make predictions about the developing world, but to study the attributes of the model and the outcome of various scenarios. The exercise, nonetheless, serves to also provide insight about the outcomes of economic and environmental systems.

Since our data were obtained from different sources with different definitions, and measurement or quantification schemes, we normalized all the variables in order to avoid the issue of incompatible units. All variables were normalized to the range 0–1. We also assumed that over time income—but not other variables—can grow beyond their initial normalization limits. Equivalently, we assumed that there exist physical upper and lower bounds on variables such as pollution.

Two simulations using the drift component of the model (with each scenario corresponding to one set of parameters) are presented below to illustrate the impacts of apparently small changes on the outcome of the economic–environmental system, with emphasis on the inverted-U relationship between income and pollution.

(a) *Base scenario*

The base scenario corresponds to a system in which income and environment are interconnected. Economic activity and food production impact the environment in their current state (i.e., proportional term) and as the level of activity increases (i.e., linear term). But there is also feedback from pollution to income and food production. The above description is representative of economic–environmental systems which are highly resource-based. For instance, where agriculture and logging, or fishing and aquaculture, are the main economic activities, the production of income and food both influence and are influenced by the ecology of the region.

As seen in Figure 2, in such a system the negative impact of economic activity on the environment brings a halt to income and food growth and eventually reverses the rising trend of pollution.

Pollution as a function of income is shown in Figure 3. In this scenario, the income–environment chart resembles an EKC. It is important however to note that, despite its shape, this relationship does not necessarily imply that income growth results in higher environmental quality (for a counter-example, see below). Rather, this exemplifies the concern raised by Arrow *et al.* (1995) that if the economic–environment interdependence results in the impedance of economic growth by environmental degradation, there is a basis for policies which are focused towards the environment, rather than towards growth.

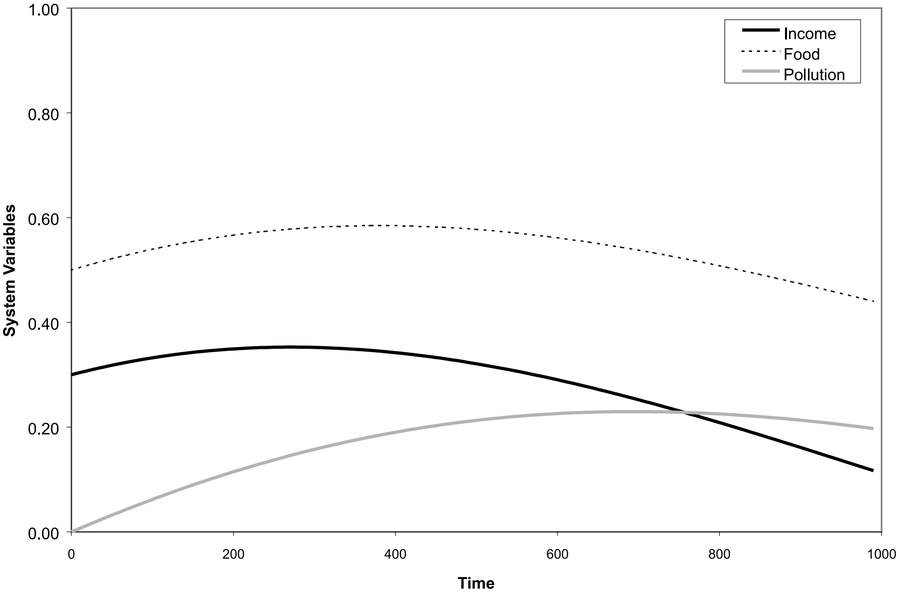

Figure 2. *The evolution of the economic and environmental variables for the base scenario. In this example, feedback from the environment to income and food production causes the process of economic growth to reverse with the increasing environmental damage due to pollution. Pollution decreases only when (harmful) economic activity has declined.*

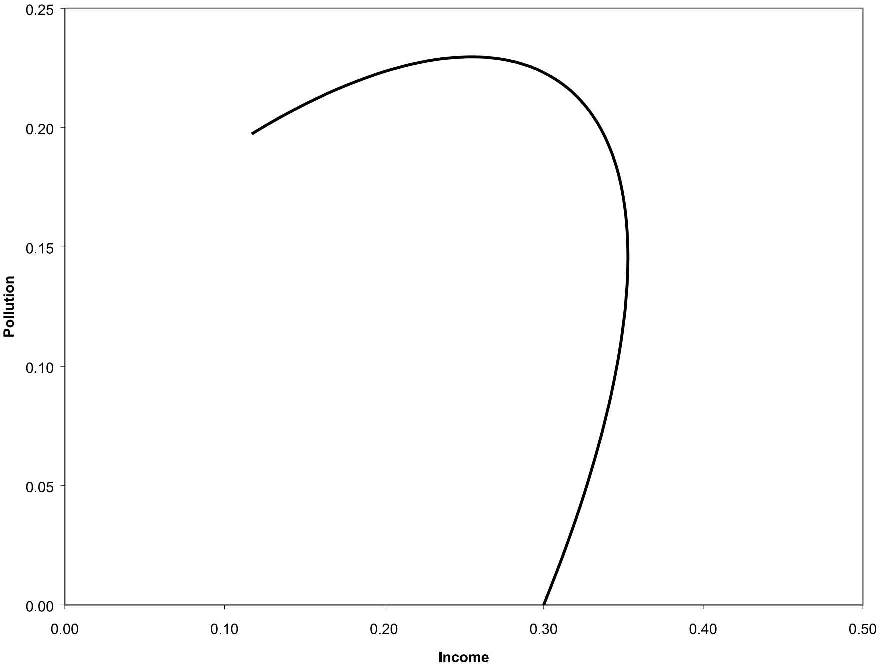

Figure 3. *Income–environment relationship for the base scenario. This relationship resembles an EKC. But, as seen in Figure 2, where economic growth and the environment are tightly coupled through negative feedback, the environment improves only when polluting activities slow down. Further, environmental degradation comes in conflict with the growth process providing a basis for environment-focused policies, confirming a concern raised by Arrow et al. (1995).*

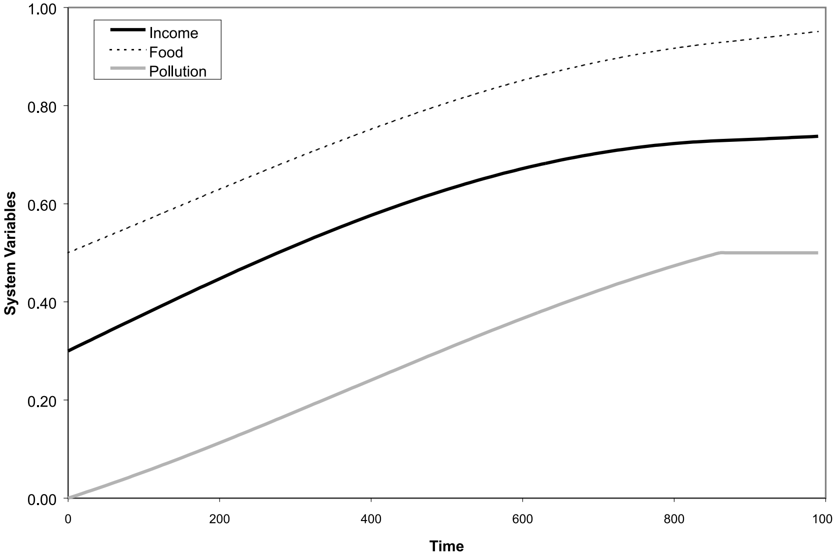

Figure 4. *The evolution of the economic and environmental variables with higher income growth rate than the base scenario. Increasing the intensity of economic activity, when the production process is inherently damaging to the environment, prevents ecological recovery which was seen in Figure 2 (see Panayotou, 1997 for another example of the importance of growth rate).*

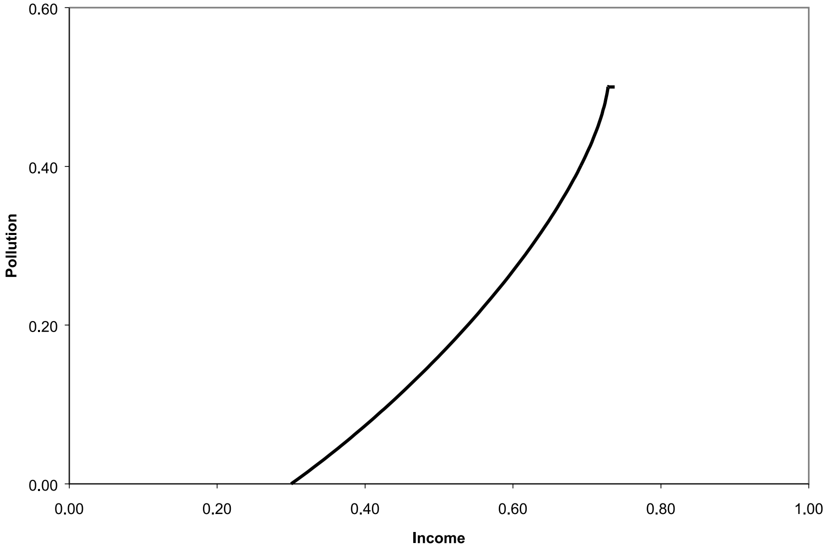

Figure 5. *Income–environment relationship after increasing the income growth rate. Higher income growth rate does not produce the same relationship as the base scenario (seen in Figure 3), when productive activity comes at the cost of increasing pollution. Therefore, the path of growth is important in addition to growth itself (see also Arrow et al., 1995; Panayotou, 1997).*

(b) *Alternative scenario: higher rate of economic growth*

In this scenario, we doubled income growth rate compared to the Base Scenario to observe whether the income–environment relationship depends on the path that is taken to reach a certain income level in addition to the level of income itself. The result for the various variables in the system and for the income–environment relationship are illustrated in Figures 4 and 5, respectively.

In this scenario the increased intensity of economic activity damages the environment to the extent that it cannot recover. The collapse of fish stock as a result of harvesting beyond the maximum sustainable rate or severe water pollution caused by industrial activity are examples of such a phenomenon in ecological–environmental systems. The comparison of the outcome to the base scenario illustrates that, when economic and environmental phenomena interact through feedbacks, the relationship between income and the environment depends on not only the level of income, but also how that income level is achieved (see Panayotou, 1997 for another example).

6. ISSUES IN APPLICATION OF THE MODEL

Having completed the description of the model and the conditions which result in an inverted-U or other specific outcomes, we briefly consider some of the practical issues which arise when a conceptual abstraction, such as this model, is applied to policy analysis. We focus this discussion on the estimation of model parameters and the quantification of model variables.

(a) *Estimation of model parameters and data requirements*

Interactions among economic and environmental variables often take place at a scale considerably smaller than a country. Even the impacts of global climate change is mediated through local conditions such as land-use and access to technology. For this reason, in our formulation, we have used examples of economic and environmental variables at the local level such as food consumption or production. This approach permits choosing the variables of the model according to their local importance and estimate their interaction parameters locally rather than using aggregate values which may not be applicable to any individual region (see Kaufmann *et al.*, 1998 for a discussion of the importance of the scale of analysis).

The parameters of the model need to be estimated locally unless there are reasons—based on the physical attributes of the system—to believe that those parameters estimated elsewhere are valid. Some parameters, such as the dose–response curve describing the relationship between air pollution and respiratory diseases, are likely to be transferable among settings. Others, such as the relationship between agricultural activity and soil erosion, change considerably from one place to another, requiring case-specific estimation.

Since data from a single site, which allow simultaneous estimation of parameters, are rare in development research, parameter values from extant literature coupled with sensitivity analysis methods can form the basis for assessing the range of possible outcomes associated with a given intervention or policy (see Nordhaus, 1994 for an example of such an analysis). This result can in turn become the basis for setting subsequent data collection priorities which would facilitate simultaneous parameter estimation.

(b) *Quantification of variables*

An inherent requirement for the application of an analytical model is the quantification of the variables of interest. This requirement in turn necessitates that a measurable and quantifiable definition for each of the variables of interest is provided, still a subject of great debate. It has only been in the recent past that the idea of quantifying ecological resilience has entered the mainstream development literature (Arrow *et al.*, 1995). Further, the measurement of the more "traditional" development variables such as income by—GNP or purchasing power parity—or health—by life expectancy, mortality, morbidity, or disability adjusted life years (DALY)—is also not universally agreed upon. In developing the model we inherently assume that its use is accompanied by a sensible measure of the variables of interest. We also emphasize that various definitions of such variables, and in turn various measurement and quantification methods, can lead to substantially different results (see, for example, Rothman, 1998 which uses consumption—rather than pollution—as a measure of environmental impact, providing an additional conceptual framework for studying the relationship between growth and the environment).

7. CONCLUSIONS

Important recent work has focused on the relationship between income and environment as the basis of policy formulation. The environmental Kuznets curves have in particular attracted a great deal of attention in development and environment policy (Beckerman, 1992; World Bank, 1992). In this article we drew attention to some of the shortcomings of this approach to environment-development analysis. We then used this critique as the basis of an alternative method which explicitly considers the dynamics of the development process and the interactions among various phenomena that determine the course of evolution of each socioeconomic or environmental variable. Our formulation therefore extends the theoretical component of the well-thought out works of de Bruyn (1997) and Panayotou (1997) that, despite pointing to some of the important factors which may influence the actual shape of the income–environment relationship, stop short of formalizing a method for representing the interrelationships and feedbacks of a complex economic–environmental system explicitly.

We introduced a restricted class of vector-valued differential equations to represent the dynamics of socioeconomic and environmental variables as they influence, or are influenced by, the process of economic growth. This formulation takes explicit account of the interrelationships among variables over time and allows for explicit incorporation of the impacts of policy or technology on the structure of an economic–environmental system in the form of model parameters. Using this model in its simplest specifications, we were able to identify the constraints on sets of variables that lead to an inverted-U pattern for any one of the variables in the system. Our analytical and numerical examinations illustrate that an inverted-U is only one possible outcome among a diverse set of relationships, and comes about under specific conditions involving not only income but also other important variables (and as a result of pair-wise association between various economic or environmental variables). By considering the possibility of a diverse set of relationships, our result contrasts with many existing works that have focused on specific patterns (such as an inverted-U pattern) for the income–environment relationship without analytically considering how such patterns are formed through the interaction of policy, technology, and ecology.

We also briefly addressed the data requirements for the application of the model. The model itself provides a framework that clarifies what data are required for meaningful projections which take into account the interactions within the system. In addition to explicitly clarifying "data gaps," our work provides an analysis tool with more flexible structure, in which the nature and the consequences of interventions can be considered at a detailed level. More specifically, by considering the feedback among various variables, it is possible to isolate the most sensitive components of the system. This facilitates prioritization of interventions, and focuses future analysis and data collection to assess the likely consequences of targeted programs.

Finally, an example of the implementation of the model at the micro-level illustrated that in analyzing economic–environmental systems, one must pay attention not only to income growth but also to the multiple variables and interactions which influence the outcome, therefore confirming an issue qualitatively raised by Arrow *et al.* (1995).

NOTES

1. See Banuri (1997) and Stern (1998) for thorough and excellent reviews of EKC literature, their assumptions, and their critiques.

2. See Kaufmann, Davidsdottir, Garnham and Pauly (1998) and Stern (1998) for a detailed discussion of the need to consider accumulated pollution in addition to emissions.

3. Note that even when panel data are used, countries are observed at different times and many occupy only small sections of the whole income range; in other words

the data are in the form of a synthetic cohort rather than a complete panel.

4. Cole *et al.* (1997) do not find statistical evidence for feedback from emissions to income. Physical evidence, however, is abundant in economic activities which are dependent on ecological systems such as timber, fisheries, and agriculture. The impacts of acid rain on forest-based activities provide another example of feedback from environment to the economy.

5. Koop and Tole (1999) overcome this problem by allowing the model parameters to vary across countries. Doing so, they find no evidence of an EKC for deforestation.

6. A more complete treatment also adds a stochastic component to Eqn. (2) to account for factors in addition to the variables in the system (such as global climate, macroeconomic effects, etc.). Eqn. (2) is then re-written as

$$d\mathbf{V}(t) = \mathbf{b}(\mathbf{V}(t), t)\,dt + \mathbf{C}(\mathbf{V}(t), t)\,d\mathbf{w}(t), \quad \mathbf{V}(t = 0) = \mathbf{V}^0,$$

where as above $\mathbf{b}(\mathbf{V}(t), t)$ is the drift vector. $\mathbf{C}(\mathbf{V}(t), t)$ is the covariance matrix and $\mathbf{w}(t)$ denotes the n-dimensional Brownian motion. For reasons of space, we limit the discussion to the drift term only. A brief treatment of the stochastic component is given in Ezzati, Singer, and Kammen (1999).

7. The parameters are in general time and state dependent and can change as a result of policy or technology.

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APPENDIX A. THE OUTCOME OF THE DRIFT PROCESS

Using the drift vector given by Eqn. (2), the system is described by the following system of ordinary differential equations (obtained from rewriting Eqn. (3)):

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} = \mathbf{b}_0(t) + \mathbf{B}(t, \mathbf{V}(t))\mathbf{V}(t) + \mathbf{D}(t, \mathbf{V}(t))\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}. \tag{A.1}$$

Rearranging this equation, we get

$$(\mathbf{I} - \mathbf{D}(t, \mathbf{V}(t)))\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} = \mathbf{b}_0(t, \mathbf{V}(t)) + \mathbf{B}(t, \mathbf{V}(t))\mathbf{V}(t), \tag{A.2}$$

where $\mathbf{I}$ is the identity matrix. Assuming that the matrix $(\mathbf{I} - \mathbf{D}(t, \mathbf{V}(t)))$ is nonsingular, we can re-write

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} = (\mathbf{I} - \mathbf{D}(t, \mathbf{V}(t)))^{-1}\mathbf{b}_0(t, \mathbf{V}(t)) + (\mathbf{I} - \mathbf{D}(t, \mathbf{V}(t)))^{-1}\mathbf{B}(t, \mathbf{V}(t))\mathbf{V}(t). \tag{A.3}$$

Denoting $(\mathbf{I} - \mathbf{D}(t, \mathbf{V}(t)))^{-1}\mathbf{b}_0(t, \mathbf{V}(t))$ by $\mathbf{h}(t, \mathbf{V}(t))$ and $(\mathbf{I} - \mathbf{D}(t, \mathbf{V}(t)))^{-1}\mathbf{B}(t, \mathbf{V}(t))$ by $\mathbf{A}(t, \mathbf{V}(t))$, we have

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} = \mathbf{A}(t, \mathbf{V}(t))\mathbf{V}(t) + \mathbf{h}(t, \mathbf{V}(t)). \tag{A.4}$$

Allowing time-varying parameters for the model is necessary to capture both the physical properties of ecological systems and the impacts of policy interventions when they modify the behavior of the system. At the same time, in economic–ecological systems where modification of the environmental impacts (such as the introduction of new technology) often involves large-scale efforts, it is reasonable to assume that the number of parameter changes are finite.

With this assumption, Eqn. (A.4) can be rewritten as a series of initial value nonhomogeneous first-order linear differential equations with constant coefficients, each having the form

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} = \mathbf{A}\mathbf{V}(t) + \mathbf{h} \tag{A.5}$$

(i.e., the equation is solved piece-wise, with each piece corresponding to one change in parameter values).

Assuming that $\mathbf{A}$ has a basis of eigenvectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m$, the matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m]$ is nonsingular. We can then diagonalize the vector $\mathbf{V}(t)$ as

$$\mathbf{V}(t) = \mathbf{X}\mathbf{Z}(t),$$

which in turn implies

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} = \mathbf{X}\frac{\mathrm{d}\mathbf{Z}(t)}{\mathrm{d}t}.$$

Eqn. (A.5) can therefore be re-written as

$$\mathbf{X}\frac{\mathrm{d}\mathbf{Z}(t)}{\mathrm{d}t} = \mathbf{AXZ}(t) + \mathbf{h}. \tag{A.6}$$

Multiplying by $\mathbf{X}^{-1}$ from the left we get

$$\frac{\mathrm{d}\mathbf{Z}(t)}{\mathrm{d}t} = \mathbf{X}^{-1}\mathbf{AXZ}(t) + \mathbf{X}^{-1}\mathbf{h}. \tag{A.7}$$

Since the columns of $\mathbf{X}$ are the eigenvectors of $\mathbf{A}$, $\mathbf{X}^{-1}\mathbf{AX}$ is a diagonal matrix with its main diagonal consisting of the eigenvalues of $\mathbf{A}$ as follows:

$$\mathbf{X}^{-1}\mathbf{AX} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda_m \end{bmatrix}. \tag{A.8}$$

Therefore the system of differential equations in Eqn. (A.7) can be decomposed in terms of its components giving

$$\frac{\mathrm{d}z_i(t)}{\mathrm{d}t} = \lambda_i z_i(t) + r_i, \tag{A.9}$$

where r_i is a linear combination of $h_1, \ldots, h_m$ (i.e., $\mathbf{r} = \mathbf{X}^{-1}\mathbf{h}$). The solution of Eqn. (A.9) is readily given by

$$z_i(t) = \mathrm{e}^{\lambda_i t}\left(\int \mathrm{e}^{-\lambda_i s} r_i \,\mathrm{d}s + k_i\right), \tag{A.10}$$

where k_i is a unique constant determined by using the initial condition of the system described by the relationship $\mathbf{V}(t=0) = \mathbf{V}^0 = \mathbf{XZ}(0)$. Since the r_i do not depend on time (the elements of $\mathbf{b}_0$ and those of the vector $\mathbf{h} = (\mathbf{I} - \mathbf{D})^{-1}\mathbf{b}_0$ are constant: thus, r_j which are linear combinations of $h_1, \ldots, h_m$ are constant), the solution given by Eqn. (A.10) can be re-written as

$$z_i(t) = -\frac{r_i}{\lambda_i} + k_i \mathrm{e}^{\lambda_i t}. \tag{A.11}$$

It follows from this solution that the vector of socioeconomic or environmental variables in the system, $\mathbf{V}(t)$, is given by a sequence of vectors of the form

$$\mathbf{V}(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \\ \vdots \\ V_m(t) \end{bmatrix} = \mathbf{XZ}(t)$$

$$= \mathbf{X}\begin{bmatrix} -\lambda_1^{-1} r_1 \\ -\lambda_2^{-1} r_2 \\ \vdots \\ -\lambda_m^{-1} r_m \end{bmatrix} + \mathbf{X}\begin{bmatrix} k_1 \mathrm{e}^{\lambda_1 t} \\ k_2 \mathrm{e}^{\lambda_2 t} \\ \vdots \\ k_m \mathrm{e}^{\lambda_m t} \end{bmatrix} \tag{A.12}$$

Table 1. *Conditions for specific outcomes of the drift component and examples of pollutants that exhibit each behavior*

Trajectory of $\mathbf{V}_i$	Conditions required for outcome
Increasing (example: per capita and aggregate carbon emissions; municipal waste)	$\sum_{j=1}^{m} X_{ij} k_j \lambda_j \mathrm{e}^{\lambda_j t} > 0 \quad \forall t$
Decreasing (example: water pollution due to lack of sanitation)	$\sum_{j=1}^{m} X_{ij} k_j \lambda_j \mathrm{e}^{\lambda_j t} < 0 \quad \forall t$
U-shaped (with turning point at $t = T$) (Example: rural indoor air pollution decreasing with a change in fuel and then increasing with rise in energy consumption)	$\sum_{j=1}^{m} X_{ij} k_j \lambda_j \mathrm{e}^{\lambda_j t} < 0 \quad \forall t < T$ $\sum_{j=1}^{m} X_{ij} k_j \lambda_j \mathrm{e}^{\lambda_j t} > 0 \quad \forall t > T$
Inverted-U-shaped (with turning point at $t = T$) (example: urban particulate matter pollution; sulfur emissions)	$\sum_{j=1}^{m} X_{ij} k_j \lambda_j \mathrm{e}^{\lambda_j t} > 0 \quad \forall t < T$ $\sum_{j=1}^{m} X_{ij} k_j \lambda_j \mathrm{e}^{\lambda_j t} < 0 \quad \forall t > T$

with initial condition $\mathbf{V}(t=0)=\mathbf{V}^0$. The columns of $\mathbf{X}$ are the eigenvectors of $\mathbf{A}$; $\lambda_1, \lambda_2, \ldots, \lambda_m$ the eigenvalues of $\mathbf{A}$; and $r_1, r_2, \ldots, r_m$ linear combinations of the system parameters.

A.1. *Conditions for an inverted-U outcome as a result of the drift process*

A variable, V_i, which strictly increases or decreases over its course, must have a positive or negative first derivative, respectively. If V_i follows an inverted-U path (i.e., if it first increases and then declines) it must have an initially positive first derivative which changes to negative at some turning point. The first derivative of the vector of socioeconomic and environmental variables in Eqn. (A.12) is given by

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}=\begin{bmatrix}\frac{\mathrm{d}V_1(t)}{\mathrm{d}t}\\ \frac{\mathrm{d}V_2(t)}{\mathrm{d}t}\\ \vdots\\ \frac{\mathrm{d}V_m(t)}{\mathrm{d}t}\end{bmatrix}=\mathbf{0}+\mathbf{X}\begin{bmatrix}k_1\lambda_1 \mathrm{e}^{\lambda_1 t}\\ k_2\lambda_2 \mathrm{e}^{\lambda_2 t}\\ \vdots k_m\lambda_m \mathrm{e}^{\lambda_m t}\end{bmatrix}$$

$$=\mathbf{X}\begin{bmatrix}k_1\lambda_1 \mathrm{e}^{\lambda_1 t}\\ k_2\lambda_2 \mathrm{e}^{\lambda_2 t}\\ \vdots\\ k_m\lambda_m \mathrm{e}^{\lambda_m t}\end{bmatrix}. \quad (\mathrm{A}.13)$$

Note that once again, when the system parameters change, piece-wise differentiation results in a derivative which consists of a sequence of vectors of this form. The treatment is identical.

The conditions for the various patterns for the variable V_i are summarized in Table 1, where, as above, the columns of $\mathbf{X}$ are the eigenvectors of matrix $\mathbf{A}$, itself determined by the matrix of the parameters of the system; λ_j are the eigenvalues of $\mathbf{A}$. Finally, pair-wise relationships between any two variables, V_i and V_j, in the system can be analyzed by considering the sign of

$$\frac{\mathrm{d}V_i}{\mathrm{d}V_j}=\frac{\mathrm{d}V_i}{\mathrm{d}t}\bigg/\frac{\mathrm{d}V_j}{\mathrm{d}t}.$$

APPENDIX B. MODEL PARAMETERS USED IN SIMULATIONS OF SECTION 5

Table 2. *Parameters for the constant term*

Parameter	Sign	Interpretation
$b_{0_{\text{income}}}$	0	No (exogenous) fixed periodic increments to income
$b_{0_{\text{food}}}$	0	No (exogenous) fixed periodic increments to food production
$b_{0_{\text{pollution}}}$	–	Environment recovers from pollution over time

The parameter estimates used in simulations of Section 5 and their sources are documented in detail in Ezzati *et al.* (1999). The following tables provide a summary of the properties of the model parameters. The actual parameter values were normalized to correspond with the normalization of the system variables described in Section 5. Matrices $\mathbf{b}_0$, $\mathbf{B}$, and $\mathbf{D}$ are from the drift equation (A.1) (see Tables 2–4).

Table 3. *Parameters for the proportional term*

Parameter	Sign	Interpretation
$B_{\text{income–income}}$	+	Positive rate of income growth
$B_{\text{income–food}}$	0	No direct effect on income from food production/consumption
$B_{\text{income–pollution}}$	–	Income grows more slowly in the presence of ecological damage
$B_{\text{food–income}}$	0	Food consumption/production remains constant at constant income
$B_{\text{food–food}}$	0	*Rate* of change in food production/consumption is not a function of its current level
$B_{\text{food–pollution}}$	0	Food production stays constant in stable ecological conditions
$B_{\text{pollution–income}}$	+	Current level of economic activity continues to cause environmental damage
$B_{\text{pollution–food}}$	+	Current level of food production continues to cause environmental damage
$B_{\text{pollution–pollution}}$	0	Rate of ecological recovery is not a function of current pollution level

Table 4. *Parameters for the linear term*

Parameter	Sign	Interpretation
$D_{\text{income–income}}$	0	No feedback effect in income growth
$D_{\text{income–food}}$	0	No direct effect on income from food production
$D_{\text{income–pollution}}$	−	Increasing pollution reduces the rate of income growth
$D_{\text{food–income}}$	+	Demand for food increases with increasing income
$D_{\text{food–food}}$	0	No self-feedback in growth of food consumption
$D_{\text{food–pollution}}$	−	Food production increases with decreasing environmental damage
$D_{\text{pollution–income}}$	+	Increasing economic activity results in increasing pollution
$D_{\text{pollution–food}}$	+	Increasing food production results in increasing environmental damage
$D_{\text{pollution–pollution}}$	0	No self-feedback in ecological recovery